Investment Analysis of Structured Credit Product

Investment analysis manual

www.linkedin.com/in/hogan-tong/ p. +86 15010199852 hogan.tong@tong-email.com

Table of Contents

Introduction of Structured Credit Product

Brief Introduction of ABS/MBS/CLO/CDO

An asset-backed security (ABS) is a security whose income payments and hence value is derived from and collateralized (or "backed") by a specified pool of underlying assets. The pool of assets is typically a group of small and illiquid assets which are unable to be sold individually. Pooling the assets into financial instruments allows them to be sold to general investors, a process called securitization, and allows the risk of investing in the underlying assets to be diversified because each security will represent a fraction of the total value of the diverse pool of underlying assets. The pools of underlying assets can include common payments from credit cards, auto loans, and mortgage loans, to esoteric cash flows from aircraft leases, royalty payments and movie revenues. Mortgage-backed security (MBS) is a specific type of ABS that pools mortgage as collateral assets. Collateral loan obligation (CLO) pools other types of loans such as credit card receivables, corporate loans, and auto loans. Collateral Debt Obligation (CDO) is more of a general term. CDO pools debts, such as corporate bonds, MBS, and esoteric credit assets as underlying assets. The boundary of CLO, CDO and ABS is unclear, the terminologies have been used interchangeably in the capital markets. They are essentially the same thing.

Most asset-backed securities (ABSs) are issued as separate tranches, or classes, of securities, which have different risks and yields. Many structures are based on sequential pay tranches, where the highest tranche receives all the payments of principal until all investors of the tranche have been paid off, then the next tranche receives the principal payments, then the next, and so on, until all investors have been paid. Investors of the lowest tranches are not likely to be fully repaid, however, and may suffer large losses.

Securitization Deal Procedure

Securitization deals are mostly done in the following procedure:

Initial deal
- Select underlying assets (Collaterals)

Pre-closing period
- Acquire ("warehouse") assets prior to the closing date with the intention of transferring them to SPV on the closing date.
- The collateral will be transferred to a temporary SPV, prepared to be sold to investors. It can also remain on issuer's book at this time, but should be treated separately from issuer's other assets.

Closing date
- In most cases, cash flows generated post closing date belong to the CDO investors.

Ramp-up period
- The SPV purchases the assets with the proceeds from the sale of debt and equity, either in a "period of time", or buy all the assets at a certain date. In Chinese mainland market it is more common to prepare the deal during the ramp-up period and purchase everything afterwards in one day.

Ramp-up date
- In some cases, the asset manager purchases all the collateral assets at ramp-up date.
- CDO notes could also be issued at ramp-up date, it makes no difference if the notes are issued at closing date or ramp-up date as long as there is a pre-determined date after which the cash flows from collaterals belong to investors.

Keys to Forecasting Cash Flow

Asset with predictable cash flows are commonly valued using the discounted cash flow approach, which can be further broke into two parts: predicting cash flows and predicting interest rates. There has been rich literature and enough models in interest rates analysis, but methods of modelling ABS cash flow are still quite limited. Predicting default and prepayments (or the probability distribution of defaults and prepayments) of pooled assets requires the most complicated math and statistics in finance, while discount rate is relatively easy to decide, i.e. required rate of return. Therefore, the investment analysis of structured products is mostly about cash flow prediction.

Better prediction usually comes with higher requirement of computation capability. A CDO with 20 names requires very little coding, while a CLO with 100,000 loans could take a day for the computer to finish Mont Carlo simulations. Therefore, we must find a method that balances accuracy and cost of resources.

In this manual, we review the bedrock of defaults and prepayments of pooled assets and establish an investment screening procedure for such asset class.

Survival Analysis Approach

Survival analysis is the mathematical method derived to predict the survival and termination of entities. It was first developed and used in biostatistics to study the survival and demise of herds. Survival analysis was introduced to ABS business in 2000.

The Basic Math

In this section we introduce the deduction of survival function, based on Li (2000). We consider an existing debt $\mathcal{B}$, the termination time of this debt is denoted as T. T can be continuous or discrete, for the simplicity we assume it is continuous. Let $\mathrm{F(t)}$ denote the probability distribution function of T,

$$\mathrm{F(t)} = \Pr(T < t), \quad t \geq 0$$

The probability density function is therefore defined as $\mathrm{f(t)} = \mathrm{F'(t)}$. And we define the **Survival Function** as,

$$\mathrm{S(t)} = 1 - \mathrm{F(t)} = \Pr(T \geq t), \quad \mathrm{t} \geq 0$$

Obviously, the probability of a debt surviving beyond its initiating time is 1 (a debt can't default the moment it was written), therefore $\mathrm{F(0)} = 0$ and $\mathrm{S(0)} = 1$. $\mathrm{S(t)}$ is the probability that the debt will survive beyond time t. Since each asset in a CDO may have unique initializing date and existing status, we need to push the survival function a little further to deduct the conditional survival probability.

$$\Pr(\mathrm{T} < \mathrm{t} + \Delta\mathrm{t} \mid \mathrm{T} \geq \mathrm{t}) \quad \mathrm{t}, \Delta\mathrm{t} > 0$$

Above is the probability of terminating before $\mathrm{t} + \Delta\mathrm{t}$, on the condition of surviving past t. This probability is used to simulate default time when the debt has already survived for a period of time. The intuition is simple, for a 10-year debt, the probability of default in 3 years should be different in year 1 and year 6. In most cases, the debtor is less likely to default at the start of the debt, but the default probability might rise as time goes because its liquidity situation might have changed after several payments into the debt.

$$\Pr(T<t+\Delta t|T\geq t) = \frac{Pr(terminate\ between\ t\ and\ t+\Delta t)}{Pr(survive\ passed\ t)} = \frac{F(t+\Delta t)-F(t)}{1-F(t)} \approx \frac{f(t)\Delta t}{1-F(t)} = -\frac{S'(t)\Delta t}{S(t)} \qquad (1)$$

$\frac{f(t)}{1-F(t)}$ is denoted as $\lambda(t)$, which is called **Hazard Rate Function**. The following deduction is a crucial step:

$$\lambda(t) = -\frac{S'(t)}{S(t)}$$

$$\Rightarrow \int_0^t \lambda(x)\,dx = \int_0^t -\frac{S'(x)}{S(x)}dx$$

$$\Rightarrow \int_0^t \lambda(x)\,dx = -\int_0^t \frac{1}{S(x)}dS(x) = -\int_0^t dln\big(S(x)\big) = \ln\big(S(0)\big) - \ln\big(S(t)\big)$$

Since $S(0) = 1$,

$$\int_0^t \lambda(x)\,dx = -ln\big(S(t)\big)$$

$$S(t) = e^{-\int_0^t \lambda(x)dx}$$

Therefore, we have,

$$\Pr(T<t+\Delta t|T\geq t) = e^{-\int_0^{t+\Delta t}\lambda(x)dx}/e^{-\int_0^t \lambda(x)dx} = e^{-\int_t^{t+\Delta t}\lambda(x)dx} \qquad (2)$$

$\Pr(T<t+\Delta t|T\geq t)$ is the key to simulate debt termination, the problem of estimating $\Pr(T<t+\Delta t|T\geq t)$ is converted to estimating the hazard rate function $\lambda(t)$.

Static Pool

Before going into model estimation, we first introduce static pool. Static Pool Analysis (also Cohort or Vintage Analysis) is the risk analysis of a pool of risks linked to a specific time period (typically the time these risks first entered the portfolio). Static pool data is commonly recorded in the following form:

		Pool construction date				
		Jan 2016	Feb 2016	March 2016	Apr 2016	May 2016
Observation date	Jan 2016	●				
	Feb 2016	●	●			
	March 2016	●	●	●		
	Apr 2016	●	●	●	●	
	May 2016	●	●	●	●	●
	Jun 2016	●	●	●	●	●
	Jul 2016	●	●	●	●	●

Suppose the static pool of loans initiated in Jan 2016 is as below, we now show how to translate it to the table above.

ID	Start Date	Initial Amount	Frequency	Terms	Interest Rate	Default Time
1	Jan 1 2016	$10,000	Month	12	3.5%	6
2	Jan 4 2016	$15,000	Month	15	3.5%	9
3	Jan 5 2016	$12,000	Month	18	3.5%	-
4	Jan 7 2016	$30,000	Month	15	3.5%	3
5	Jan 11 2016	$30,000	Month	10	3.5%	-
6	Jan 3 2016	$12,000	Month	20	3.5%	-
7	Jan 6 2016	$10,000	Month	18	3.5%	16
8	Jan 6 2016	$10,000	Month	18	3.5%	-

In Jan 2016, no loan defaulted because they were just written, therefore, the up-left dot in the table should be 0%. Then we turn to the next observation date, which is Feb 2016, and noticed still no default occurred, so it should also be 0%. In March 2016, Loan 4 defaulted and so 12.5% of the initial number of loans defaulted. We do the same thing for the rest of observation dates, and got:

Table 1 Sample Static Pool

		Pool construction date				
Observation date		Jan 2016	Feb 2016	March 2016	Apr 2016	May 2016
	Jan 2016	**0%**				
	Feb 2016	0%	●			
	March 2016	12.5%	●	●		
	Apr 2016	12.5%	●	●	●	
	May 2016	12.5%	●	●	●	●
	Jun 2016	25%	●	●	●	●
	Jul 2016	25%	●	●	●	●
	Aug 2016	25%	●	●	●	●
	Sep 2016	37.5%	●	●	●	●
	Oct 2016	37.5%	●	●	●	●
	Nov 2016	37.5%	●	●	●	●
	Dec 2016	37.5%	●	●	●	●

We can use either number of defaults or amount of defaults to calculate the static pool data. Other columns are constructed in the same way.

Estimate Hazard Rate Function

We have established that $\lambda(t)\Delta t$ is the conditional probability of surviving passed t but terminated at $t + \Delta t$, we denote this probability as $P_t^{t+\Delta t}$. For reasons we will see later in this article, we derive the discrete form of conditional termination probability:

$$P_n^{n+1} \qquad n = 0,1,2,\ldots\ldots$$

If a debt terminated at $n+1$, that could happen in two scenarios: it didn't survive to n, or it has survived n periods but terminated the next period. The probability of terminating at $n+1$ is the sum of the probabilities of the two scenarios. Therefore,

$$P_0^{n+1} = P_0^n + (1 - P_0^n)P_n^{n+1}$$

$$\Rightarrow P_n^{n+1} = \frac{P_0^{n+1} - P_0^n}{(1 - P_0^n)} \qquad (3)$$

The logic is simple, if we get $\{P_0^n\}_{n=1,2,\ldots}$, we can derive $\{P_n^{n+1}\}_{n=1,2,\ldots}$, and use Equation (2) to derive $\lambda(t)$. It should be intuitive that $\lambda(t)$ would be a staircase function when P_n^{n+1} is discrete. The next step is to estimate $\{P_0^n\}_{n=1,2,\ldots}$, which is actually quite simple with static pool data. For example, in the Jan 2016 pool of Tabel 1, P_0^3=12.5%, $P_0^6 = 25\%$.

Plot P_n^{n+1} against n, we get the **credit curve**. We can conduct stress test by manipulating the credit curve.

Cause-specific Hazard Rate

We have assumed that only there is only one cause for debt's termination, but in reality there are two: default and prepayment. We construct two static pool tables, one for default and the other one for prepayment. Using the method above, we can get the hazard rate function for each cause of termination, we denote the hazard rate function of default as $\lambda_D(t)$, and hazard rate function of prepayment as $\lambda_P(t)$. As shown by Equation (1), the conditional termination probability is therefore $(\lambda_D(t) + \lambda_P(t))\Delta t$, which gives us the survival function with two causes of termination,

$$\mathrm{S}_{D,P}(\mathrm{t}) = e^{-\int_0^t \lambda_D(x)+\lambda_P(x)dx} \qquad (4)$$

We use $\mathcal{H}$ to denote cause of termination, default (D) and prepayment(P), as Kalbfleisch & Prentice (1979) showed,

$$F_D(t) = Pr(T < t, \mathcal{H} = \mathrm{D}) = \int_0^t \lambda_D(s)\mathrm{S}_{D,P}(\mathrm{s})\mathrm{ds} = \int_0^t \lambda_D(s)e^{-\int_0^s \lambda_D(x)+\lambda_P(x)dx}\mathrm{ds}$$

Therefore,

$$Pr(t_2 < T < t_2 + \Delta t, \mathcal{H} = \mathrm{D} | \mathrm{T} \geq t_1) = Pr(T \geq t_2 | T \geq t_1) \times Pr(t_2 < T < t_2 + \Delta t, \mathcal{H} = D | T \geq t_2) \qquad t_2 > t_1$$

$$= Pr(T \geq t_2 | T \geq t_1) \times Pr(t_2 < T < t_2 + \Delta t | T \geq t_2) \times Pr(\mathcal{H} = D | t_2 < T < t_2 + \Delta t)$$

$$= Pr(T < t_2 + \Delta t | T \geq t_1) \times Pr(\mathcal{H} = D | t_2 < T < t_2 + \Delta t)$$

$$= e^{-\int_{t_1}^{t_2} \lambda_D(x)+\lambda_P(x)dx} \times \frac{\lambda_D(t_2)}{\lambda_D(t_2)+\lambda_P(t_2)} \times \Delta t \qquad (4)$$

Equation (4) is the foundation of simulating cause of termination. The first term of the right-hand side calculates the probability of termination at t_2, and the second term calculates the probability of default on the condition of terminating at t_2.

Simulating Termination Time

Termination time can be simulated using the following method. Consider a debt that has survived for t_1. Generate a random value p from uniform distribution U(0,1) and plug it into Equation (2) to get the termination time t_2

$$p = e^{-\int_{t_1}^{t_2} \lambda(x)dx}$$

Sample codes in Python can be found in Appendix B.

Dealing with Correlation

> *Static pool data has reflected asset correlation, so it's natural to ask the question: why bother to separate asset correlation from the rest of the model? The answer is stressed test. Separating asset correlation as an independent factor will enable us to push the economic condition to extreme scenarios where asset correlations dramatically increase.*

Asset Correlation

Asset correlation is an important issue that needs to be addressed in the model. The defaults of assets in the pool are likely to be correlated through some mutual factors. Taking RMBS as an example. An RMBS consists of numerous residential mortgages, which are embedded with a put option on the residential house. If the housing price falls below the balance of the loan, debtors may intentionally default on the loan (execute the put option) and let the bank take the house. Since housing prices tend to move in the same direction within an area, if one mortgage defaults, the other mortgages from the same area may default as well, thus create a positive correlation among assets. Same goes for prepayment. If interest rate drops, every loan in the pool would be more likely to prepay because debtors can borrow at a lower rate to refinance their current mortgages.

The Basic Idea of Copula Method

As stated by Bund et al. (2006), when it comes to estimating the correlation of default, the biggest headache is lack of historical tracking data, which brings expert opinion into the model. For instance, suppose we are estimating the correlation between two groups of people's debts, two problems arise. First, the two groups are very likely to have inadequate borrowing record as samples in regression models. Second, even if they do, gathering data would be a huge work load that requires the help from specialized agencies, which are extremely rare (and desperately needed!) in most parts of the world, especially in emerging markets.

We now show an easy but less mathematically logical way of modelling asset correlation proposed by Li (2000), which is very commonly used in both academics and industry. In previous sections we showed the method of simulating termination time, which is carried out by simulate the termination probability. Simulating termination time that incorporates asset correlation is simply done by simulating correlated termination probabilities. Instead of independently draw probabilities from U(0,1), we now use the following equation.

$$p_i = N(\sqrt{\rho}\mu + \sqrt{1-\rho}\omega_i) \tag{5}$$

p_i is the simulated termination probability for asset i. $\mu \sim N(0,1)$ is a common factor across all assets, $\omega_i \sim N(0,1)$ is independently generated for each asset, therefore $\sqrt{\rho}\mu + \sqrt{1-\rho}\omega_i$ is under standard normal distribution. Plug p_i into

Equation (2) and we get the correlation-incorporated termination probability of each asset. We assume ρ is known and stressed test can be conducted by manipulating ρ.

Correlation Estimation

Estimating correlation of termination time is almost mission impossible because it is impossible to a) Gather sufficient data on termination behavior in less regulated area such as consumer credits and financial leasing, and 2) Obtain the historical data of termination time of one asset (once it terminated, it's gone, and there won't be any further data). A commonly used way to fix this problem is to use indexes that have sufficient tracking records as proxies and use the correlation of proxies as ρ. For example, for consumer credit assets, we can do the following.

I. Identify a proper proxy of consumer credit quality from macroeconomics indexes, e.g., average personal leverage ratio Lev_t, and normalize it.

II. Identify a major factor that drives Lev_t, e.g. housing price index HPI_t, and normalize it. Manipulate HPI_t in a way that the higher HPI_t is, the better credit quality. Obviously, rising housing price will enhance consumers' credit quality.

III. Estimate correlation of Lev_t and HPI_t, and use the result as ρ in Equation (5). Note that μ can now be expressed as normalized expected housing price, which is based on expert opinion or other macroeconomic predictions.

Our hope is that correlation between proxy of assets' credit quality and macro economical trend has something to do with termination correlation. This method is not strictly logical in the minds of statisticians and mathematicians, but it is easy to conduct stressed test as we can manipulate μ in Equation (5).

Investment Analysis Procedure

Step 1: Cash Flow Analysis

Based on the discussion above, we now summarize the cash flow analysis procedure of structured credit products in the chart in Appendix A.

Step 2: Valuation Analysis

Build a waterfall model with Excel or coded it in fancier scripts like Python or Matlab if you want to explore more flexibility and make it easier to maintain the model. The following steps are recommended.

I. Calculate debt service coverage ratio (DSCR) for the tranche under analysis and decide if the DSCR is within the comfort zone.

II. Calculate IRR and compare it to the required rate of return to decide whether the investment opportunity is worth further action.

III. Manipulate cumulative default/prepayment rate and the shape of hazard rate curve, to conduct stressed test.

The sample code of waterfall model in Python is provided in Appendix B.

References

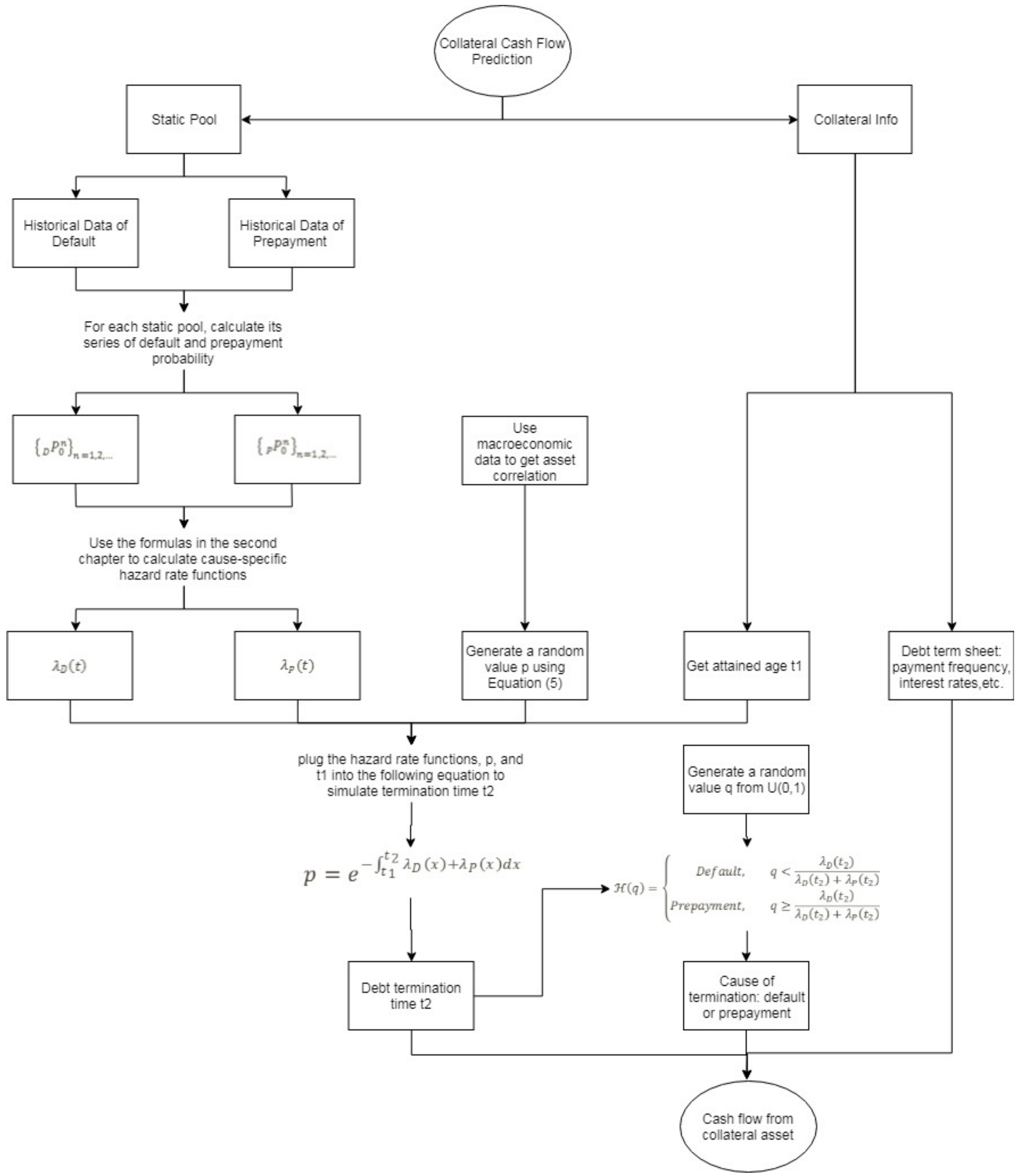
Li, D. X. (2000). On Default Correlation: A Copula Function Approach. The Journal of Fixed Income, 9(4), 43-54.

Prentice, R. L., & Kalbfleisch, J. D. (1979). Hazard rate models with covariates. Biometrics, 35(1), 25-39.

Bund, S. et al. (2006). Fitch: European Consumer ABS Rating Criteria. ABS Europe Criteria Report.

Appendix

Appendix A



Appendix B